Detecting edges in high order methods for hyperbolic conservation laws

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The detection of discontinuities in the numerical solution of a given conservation law is an important tool to handle shocks or discontinuous initial data appropriately.
Up to now there exist various approaches to find discontinuities in a numerical solution. We will focus on an edge detector based on the conjugated Fourier partial sum which has shown excellent behaviour in the context of spectral methods. This detector is based on the property of the one-dimensional conjugated Fourier partial sum of a function f to converge pointwise to the jump height of f, which is often referred to as concentration property. To accelerate the convergence rate one can exploit the fact that the conjugated Fourier partial sums can be written as a convolution $\mathcal{S}_n f(x) = f * \widetilde{D}_n(x) = \int f(t)\widetilde{D}_n(x-t)\,\mathrm{d}t$ with the conjugated Dirichlet kernel $\widetilde{D}_n$. Considering generalized kernels $\widetilde{K}_n$ with similar properties as the Dirichlet kernel as propsed in [1], it can be proven that for such generalized conjugated partial sums $\mathcal{S}^{\widetilde{K}_n} f(x) := f * \widetilde{K}_n(x)$ the concentration property holds, either. The convergence rate for this generalized sums is improved away from the discontinuity, hence an efficient edge detection with less Fourier coefficients is possible. This has been validated in several testcases in the context of spectral methods in one dimension and extended to the quasi-two-dimensional approach (with one variable fixed) in [2].
We will show that this edge detection technique can be extended to the fully two-dimensional case considering the (generalized) conjugated Fourier partial sums in two variables, where the partial sums now converge to the jumps in the mixed partial derivatives (compare [4] for the classical conjugated partial sums). Furthermore, we give different approaches to apply this edge detector efficiently in the context of general high order methods where nodal or modal coefficients are given. In the special case of the Spectral-Difference- or Discontinuous-Galerkin-method on triangles [3], a direct formulation to compute the Fourier coefficients from given modal coefficients is presented. Several testcases using these methods show that the Fourier-based detector yields superior results compared to a common coefficient-based shock indicator.

References

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