# Fast time implicit discontinuous Galerkin method for the compressible Navier-Stokes equations

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In this study, we investigate an efficient time integration technique for solving the compressible Navier-Stokes equations using a high-order discontinuous Galerkin method. We use implicit Runge-Kutta schemes for the time integration [1] associated to a Jacobian-free (JF) Newton-Krylov algorithm for the solution to the linear system at each stage. Implicit time integration is known to resolve the problem of strong restriction on the time step due to the so-called Courant-Friedrichs-Levy (CFL) condition for stability of the discontinuous Galerkin scheme associated to an explicit time discretization [2]. The discretization of the parabolic terms requires an additional stability constraint which becomes more restrictive than the CFL condition for flow regions with small cell Reynolds number. Implicit solvers are thus needed with this type of flows even at moderate CFL values as required for time-dependent simulations. However, the shortcoming of these methods is the extremely high computational cost induced by the large number of degrees of freedom (DOFs) in practical applications.

A robust and fast time integration procedure for an implicit-explicit formulation of a DG discretization has been introduced in [3, 4]. The method uses a time implicit discretization for the viscous terms, while convective terms are treated explicitly. The method is called the SIMP$p_s$ method and consists in a hierarchy of methods. It is based on a reduced communication between elements which share a common interface. This assumption follows from the natural hierarchy of the basis of the discrete function space: the communication between elements is assumed to be dominated by the coupling between DOFs associated to the components of the basis of order lower than a given degree $p_s$. After solving an implicit problem of reduced size for the low-order DOFs only, a local reconstruction, based on the stencil of the DG discretization, is used for evaluating higher-order DOFs. Numerical experiments on nonlinear convection-diffusion equations have highlighted better performances of the SIMP$p_s$ method in terms of CPU time when compared to a full implicit discretization of the viscous terms. This simplification also keeps the locality, accuracy and stability properties of the original method.

The objective of the present study is to extend the SIMP$p_s$ method to time-dependent compressible Navier-Stokes equations. We introduce an algorithm that associates the simplification of the implicit problem with the JF method. The new method is called the JF-SIMP$p_s$ method and allows to reduce the memory requirements and operation counts by avoiding the complete construction of the Jacobian matrix and solving a problem of reduced size. We will also consider the implicitation of the convection terms to guaranty a unit CFL number condition required for accurate computations of time-dependent solutions. Numerical experiments for the resolution of time-dependent flow problems in two space dimensions will be presented to assess the performances of the method.

# References


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