A low order and costly way to solve advection
that provides better results than MUSCL inexpensive limiters

Bernard Rebourcet
CEA-DAM, DIF, F-91296 Arpajon, France
bernard.rebourcet@cea.fr

• Introduction

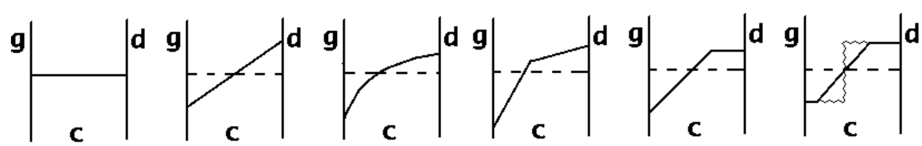
Numerous methods have been studied to solve advection equation. None the less some uncertainties remain especially when one deals with: splitted algorithm where advection is done separatly from the specific physical system (splitted ALE), and for unstructured meshes where lay tens of coupled unknowns.

This is the case of problems arisen in ICF capsule simulation (see J.Cheng HONOM 2011) where working cliffs occur. Because those flows are multimaterial the data polynomial approximation is of limited order. Furthermore the splitting between the specific equations of the model and the advection, that can be considered as a geometric phase, leads to two disctinct numerical strategies.

• The present work

The present work is the union of two concerns: about limiters properties and shape in the MUSCL framework, and the complexification of the initial problem by adding a dimension in order to advect a curve instead of a scalar quantity.

Several studies have been undertaken on how to locally build a high order representation of cell centered data for finite difference schemes. They are summarized in figure below. One wants to



Data reconstruction, from left to righ:
donor-cell, linear , PPM, double-slope, plateau-slope and discontinuous.

extend the representation sets above to shapes that allow several slopes and even discontinuity.

◇ A tautology

We consider a scalar quantity $q(t,\boldsymbol{x})$ in $\mathbb{R}^n$, $n=1,2,3$ that obeys the evolution equation:

$$\partial_t q + \boldsymbol{u}(t,\boldsymbol{x})\boldsymbol{\nabla}_{\boldsymbol{x}} q + \boldsymbol{\nabla}_{\boldsymbol{x}}.\boldsymbol{F}(t,\boldsymbol{x},q) + R(t,\boldsymbol{x},q) = S(t,\boldsymbol{x}) \tag{1}$$

associated to suitable intial and boundary conditions, with $\boldsymbol{u}$ thevelocity field, $\boldsymbol{F}$ a flux, R a production term, S a source. A splitted "lagrange-advection" algorithm solves the two equations system:

$$\partial_t q + \boldsymbol{\nabla}_{\boldsymbol{x}}.\boldsymbol{F}(t,\boldsymbol{x},q) + R(t,\boldsymbol{x},q) = S(t,\boldsymbol{x}) \tag{2}$$

$$\partial_t q + \boldsymbol{u}.\boldsymbol{\nabla}_{\boldsymbol{x}} q = 0 \tag{3}$$

In the fisrt lies the physics and the second one, $\dfrac{dq}{dt} = 0$, rules the transfert from one frame to another.The profile of q in space evolves as a surface $\Gamma(t,\boldsymbol{x},q) \equiv 0$ with:

$$\partial_t \Gamma dt + \boldsymbol{\nabla}_{\boldsymbol{x}} \Gamma d\boldsymbol{x} + \partial_q \Gamma dq = 0 \tag{4}$$

The whole system then becomes:

$$\partial_t q = \mathcal{S}(t, \boldsymbol{x}, q) \tag{5}$$

$$\partial_t \Gamma + \boldsymbol{u}.\boldsymbol{\nabla}_{\boldsymbol{x}} \Gamma + \mathcal{S}(t, \boldsymbol{x}, q) \partial_q \Gamma = 0 \tag{6}$$

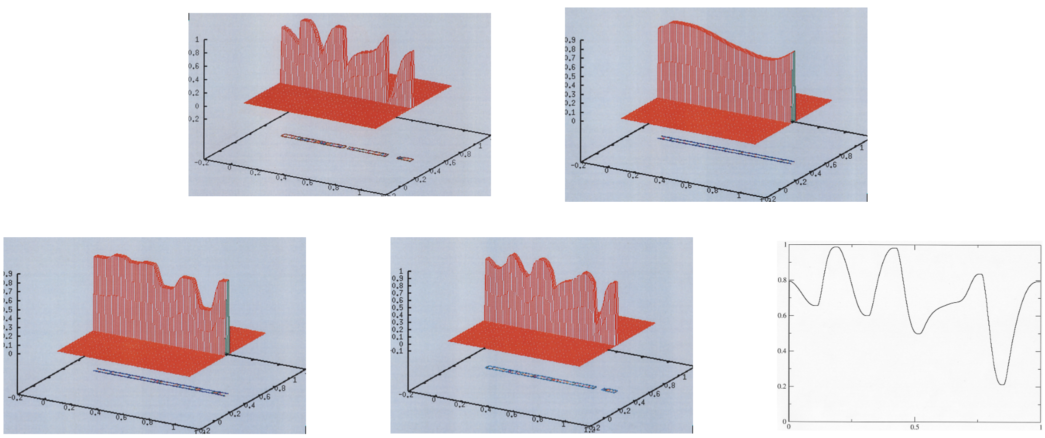
We get an advection problem with $(n+1)$ dimensions where the additionnal "velocity" comes from the resolution of the Lagrange phase.

• A Plug in game - Interface Construction

Based on calculation of lines or surfaces intersections, "Interface Construction" algorithm is expensive for standard interface. Our choice is guided by commodity because we possess it - Youngs algorithm (1982) - and that mitigates our sin of curiosity.

• Illustrations - 1D problem

One translates a periodic profile previously defined by Harten, cited by Lagoutiere (2011), with a uniform velocity. Results are obtained for a cfl number of 0.4. Space discretisation is 100 for MUSCL methods and 100×50 for ADVIC. Computations where performed on a 2D periodic domain during 200 periods (24000 cycles).

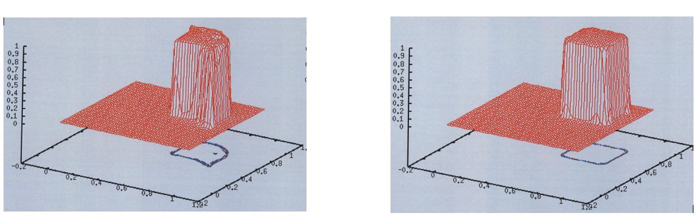


Initial shape - Van Leer (vla) - Superbee - ADVIC - vla fine

• Comments: the use of more and more compressive schemes does not alleviate the deformation of the shape along time. On the contrary ADVIC while submitted to spatial smoothing preserve the essential of the initial conditions. The counterpart is the huge amount of computational cost - almost a ratio of one hundred. To complete this picture one draws the shape obtained with *vla* on a 500 cells grid. The results is quite closed to ADVIC, the computational time is 25 greater than from a 100 cells run. As a "*lifevest*" one grabs, the lagrangian phase could be operated on a coarser grid compensating the inflationist advection phase.

• Illustrations - 2D problem

One proceed a 45° translation of a parallepipedic shape with a cfl of 0.4 during 50 cycles.



ADI and unsplit ADVIC 3D-advection.
The need of a multidimensional
flux computation
with ADVIC is correlated
to the better rendering
of stiff front.

- **Conclusion**

The goal of that work was to go-see the possibility of considering a local multi slope approximation by a posteriori reconstruction. A way to do that is to add a dimension and transform an advection problem in $\mathbb{R}^n$ in one in $\mathbb{R}^{n+1}$. The computational effort is related to the geometry based algorithm we have chosen while the results obtained meet our expectation.