A comparison of time integration schemes for DG discretizations of unsteady 3D compressible flows

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ABSTRACT

We consider the time dependent three dimensional compressible Navier-Stokes equations and their discretization using discontinuous Galerkin (DG) methods. For wall bounded flows, the boundary layer leads to extremely fine cells, meaning that the use of implicit time integration scheme becomes attractive. Here, we use ESDIRK and Rosenbrock methods, where the appearing linear and nonlinear equation systems are solved using right preconditioned Jacobian-Free Newton-Krylov schemes [5]. As a baseline scheme, these are compared to standard explicit Runge-Kutta schemes and to a type of predictor-corrector schemes that allows local time stepping for DG methods [3].

For implicit schemes, the core difficulty is to find a preconditioner for the block systems that is efficient and uses as little storage as possible, since for DG methods, the size of the blocks is much larger than for finite volume schemes. Furthermore, it should perform well in parallel. The DG method we consider is the polymorphic modal-nodal scheme of Gassner et. al., which uses a modal basis for the representation of the solution [4]. Thus, we suggest the ROBO-SGS preconditioner [1], an SGS method using reduced order offdiagonal blocks, which we find to be significantly better than ILU or the multilevel ILU of Persson et. al. [6]. These reduced blocks can be obtained in a straight forward manner due to the hierarchical basis.

Furthermore, the choice of tolerances in the adaptive time integration scheme, the Newton method and GMRES is an important point. Using embedded error estimators and inexact Newton schemes, where the tolerances are given by the strategy of Eisenstat and Walker [2], we obtain an efficient and accurate time integration scheme that has a good strong parallel scaling. This will be demonstrated by corresponding numerical results.

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