# Discontinuous Galerkin discretization of the Reynolds-averaged Navier-Stokes equations with the shear-stress transport model

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We consider the development of Discontinuous Galerkin (DG) methods for the numerical approximation of the Reynolds-Averaged-Navier-Stokes (RANS) equations with the shear-stress transport (SST) model by Menter [5]. This turbulence model [4] is based on a blending of the Wilcox $k$-$\omega$ model used near the wall and the $k$-$\epsilon$ model used in the rest of the domain. This model combines the good near-wall behaviour of the $k$-$\omega$ model and the free-stream independence of the $k$-$\epsilon$ model [3]. The switch between both models should be at about 50% of the boundary-layer. To achieve a gradual change the blending functions depend on the flow solution and on the distance to the nearest wall. For the computation of the distance of each quadrature point in the domain to the nearest of the curved, piecewise polynomial wall boundaries, we use a stabilized continuous finite element (FE) discretization of the eikonal equation, similar to [2]. Furthermore, we use a wall boundary condition for the dissipation rate $\omega$ based on the projection of the analytic near wall behavior of $\omega$ onto the discrete ansatz space of the DG discretization. One of the major problems of a higher-order discretization of a $k$-$\omega$ based turbulence model is the behavior of the turbulent kinetic energy $k$ at the boundary layer edge. In this typically underresolved part the $k$ solution has a vary rapid change in its slope leading to an undershoot and numerical oscillations. The SST model and in particular the blending functions involved are more sensitive to unphysical behaviors of $k$ than the $k$-$\omega$ model. Therefore, we introduce an artificial viscosity to the discretization of the turbulence kinetic energy ($k$-)equation to suppress oscillations of $k$ near the boundary layer edge.

The DG discretization of the RANS equations with the SST model is demonstrated for turbulent flows past a flat plate and the RAE2822 airfoil (Cases 9 and 10). The results are compared to the underlying $k$-$\omega$ model and experimental data [1]. In all test cases the results for the SST model are closer to the experiment than for the $k$-$\omega$ model. As an example, in the following we show the $c_p$-distribution over the RAE2822

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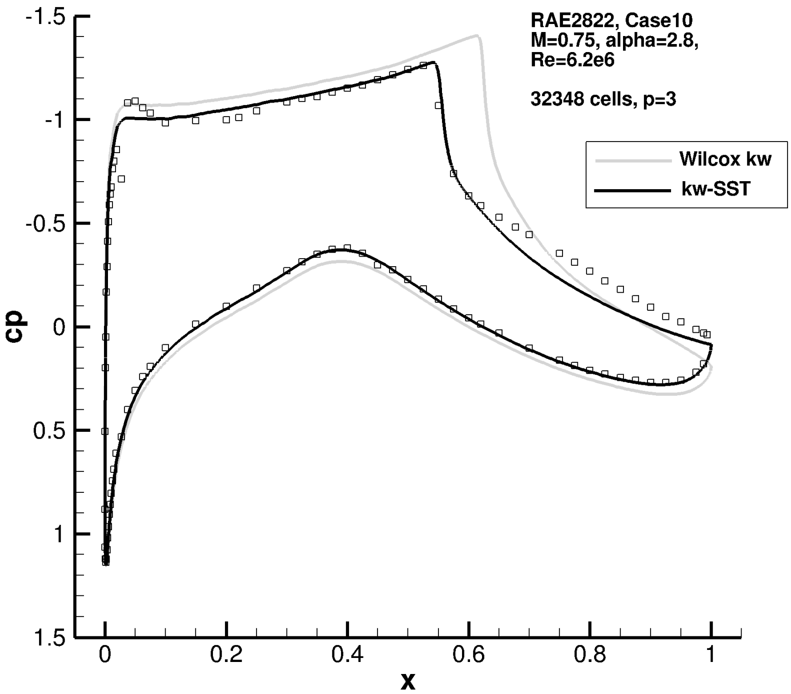
airfoil for Case 10 (Ma $= 0.75$, $\alpha = 2.8°$, Re $= 6.2 \cdot 10^6$). We see that the SST model captures the shock at the right postion, whereas the $k$-$\omega$ model locates the shock too much downstream.




## References


[1] P. H. Cook, M. A. McDonald, and M. C. P. Firmin. Aerofoil RAE 2822 - pressure distributions, and boundary layer and wake measurements. Experimental Data Base for Computer Program Assessment, AGARD Report AR 138, 1979.

[2] C.-B. Liu, P. Nithiarasu, and P. G. Tucker. Wall distance calculation using the Eikonal/Hamilton-Jacobi equations on unstructured meshes. *Engineering Computations: International Journal for Computer-Aided Engineering and Software*, 27(5):645–657, 2010.

[3] F. R. Menter. Influence of freestream values on k-$\omega$ turbulence model predictions. *AIAA Journal*, 30(6):1657–1659, 1992.

[4] F. R. Menter, M. Kuntz, and R. Langtry. Ten years of industrial experience with the SST turbulence model. In Kemal Hanjalić, Y. Nagano, and M. J. Tummers, editors, *Turbulence, Heat and Mass Transfer 4: Proceedings of the Fourth International Symposium on Turbulence, Heat and Mass Transfer, Antalya, Turkey, 12-17 October, 2003*, pages 625–632. Begell House, Inc., 2003.

[5] S. Schoenawa and R. Hartmann. Discontinuous Galerkin discretization of the Reynolds-averaged Navier-Stokes equations with the shear-stress transport model. In preparation, 2013.