A High-Order Discontinuous Galerkin Method for Fluid-Structure Interaction With Efficient Implicit-Explicit Time Stepping

Per-Olof Persson* and Bradley Froehle†

University of California, Berkeley, Berkeley, CA 94720-3840, U.S.A.

I. Introduction

We present a high-order DG formulation for the Navier-Stokes equations coupled to a finite element model of a non-linear membrane. Many approaches have been suggested for the simulation of fluid-structure interaction,[1,2] and a common way to treat the deformable domains is the use of Arbitrary Lagrangian Eulerian (ALE) methods.[3–5] In these efforts the discretization on the deformable domain is carried out on a deforming grid and thus the metric changes over time.

For the non-linear membrane model we use a continuous Galerkin discretization, integrated in time simultaneously with the DG discretization. The forces from the fluid are applied to the membrane, and the membrane displacements provide the deformation of the fluid domain. We note that this monolithic treatment provides a time-accurate coupling, unlike other approaches where the fluid and the structure are integrated separately, and forces and displacements are only transferred at the end of each timestep.

In previous work,[6] we solved a similar problem using an explicit Runge-Kutta time integrator. While this approach is simple and does not require any coupling matrices, it may introduce undesirable timestep restrictions. However, a fully implicit time integrator would require forming not only the Jacobian matrices for the fluid and the structure problems, but also the couplings between them.

Here, we demonstrate how implicit-explicit Runge-Kutta methods[7] can be used to avoid solving the fully coupled system, with arbitrarily high orders of accuracy in time. We use both the explicit and the implicit coefficients of the schemes to form a stage predictor for the force from the fluid applied to the membrane.[8] This decouples the two implicit problems for the fluid and the structure, respectively, which can be solved using standard implicit solvers.

The spatial discretization is a standard unstructured-mesh nodal discontinuous Galerkin method[9] with numerical fluxes according to the method by Roe[10] and the compact discontinuous Galerkin (CDG) method.[11] The deforming domain is handled by the mapping-based approach presented in Ref. 5. A standard neo-Hookean membrane model is used[12] with viscous, fluid-like, dissipation. We use various schemes for the mesh deformation, ranging from simple Radial Basis Functions to fully non-linear elasticity schemes.[13] The temporal discretization is based on high-order, explicit first stage, diagonally implicit Runge-Kutta (ESDIRK) methods for both the fluid and the structure. These also define the stage-predictions for the fluid forces, which we use to decouple the implicit solution of the fluid and the structure.[8] We explore the Runge-Kutta coefficients from Kennedy & Carpenter,[7] obtaining third, fourth, and fifth order convergence in time using their ARK3, ARK4, and ARK5 methods.

We verify the high-order accuracy of the scheme using a test problem of a heaving and pitching NACA airfoil in a laminar flow, subject to a simple smooth heaving motion. We also show several examples of fully coupled fluid-membrane simulations under various flow conditions.

*Assistant Professor, Department of Mathematics, University of California, Berkeley, Berkeley CA 94720-3840. E-mail: `persson@berkeley.edu`.

†Ph.D. Candidate, Department of Mathematics, University of California, Berkeley, Berkeley CA 94720-3840. E-mail: `bfroehle@berkeley.edu`.

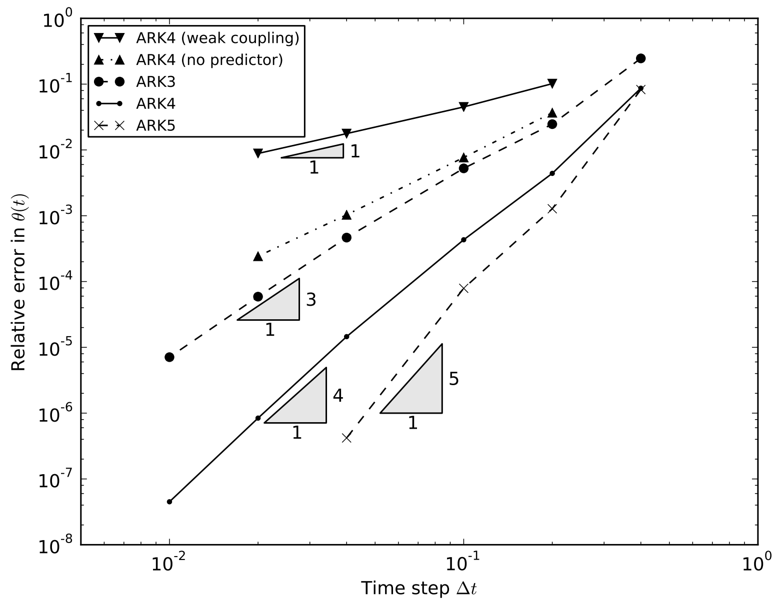

Figure 1. Convergence study of a model problem of a heaving and pitching airfoil with a torsional spring. The pivot location of the airfoil is moved upwards between time $t = 0$ and $t = 1$. We study the convergence of the angle of attack $\theta(t)$ for a range of schemes and timesteps. The figure shows the relative error in angle of attack, $||\theta(t) - \theta_{exact}(t)||_\infty / ||\theta_{exact}(t)||_\infty$, as a function of timestep. ARK3, ARK4, and ARK5 refer to the methods of the same name in Kennedy et al.[7] They are third, fourth and fifth order additive Runge-Kutta methods which have 4, 6 and 8 stages respectively. For ARK4 (weak-coupling), we integrated the fluid and structure independently on each timestep, using an implicit fourth order method. For ARK4 (no predictor) we neglected to use a predictor for the force from the fluid applied to the structure.

References

[1]Yosibash, Z., Kirby, R. M., Myers, K., Szabó, B., and Karniadakis, G., "High-Order Finite Elements for Fluid-Structure Interaction Problems," *44th AIAA/ASME/ASCE/AHS Structures, Structural Dynamics, and Materials Conference, Norfolk, Virginia*, April 2003, AIAA-2003-1729.

[2]Geuzaine, P., Grandmont, C., and Farhat, C., "Design and analysis of ALE schemes with provable second-order time-accuracy for inviscid and viscous flow simulations," *J. Comput. Phys.*, Vol. 191, No. 1, 2003, pp. 206–227.

[3]Farhat, C. and Geuzaine, P., "Design and analysis of robust ALE time-integrators for the solution of unsteady flow problems on moving grids," *Comput. Methods Appl. Mech. Engrg.*, Vol. 193, No. 39-41, 2004, pp. 4073–4095.

[4]Lomtev, I., Kirby, R. M., and Karniadakis, G. E., "A discontinuous Galerkin ALE method for compressible viscous flows in moving domains," *J. Comput. Phys.*, Vol. 155, No. 1, 1999, pp. 128–159.

[5]Persson, P.-O., Bonet, J., and Peraire, J., "Discontinuous Galerkin solution of the Navier-Stokes equations on deformable domains," *Comput. Methods Appl. Mech. Engrg.*, Vol. 198, 2009, pp. 1585–1595.

[6]Persson, P.-O., Peraire, J., and Bonet, J., "A High Order Discontinuous Galerkin Method for Fluid-Structure Interaction," *18th AIAA Computational Fluid Dynamics Conference, Miami, Florida*, 2007, AIAA-2007-4327.

[7]Kennedy, C. A. and Carpenter, M. H., "Additive Runge-Kutta schemes for convection-diffusion-reaction equations," *Appl. Numer. Math.*, Vol. 44, No. 1-2, 2003, pp. 139–181.

[8]van Zuijlen, A., de Boer, A., and Bijl, H., "Higher-order time integration through smooth mesh deformation for 3D fluid-structure interaction simulations," *J. Comput. Phys.*, Vol. 224, 2007, pp. 414–430.

[9]Peraire, J. and Persson, P.-O., *Adaptive High-Order Methods in Computational Fluid Dynamics*, Vol. 2 of *Advances in CFD*, chap. 5 – High-Order Discontinuous Galerkin Methods for CFD, World Scientific Publishing Co., 2011.

[10]Roe, P. L., "Approximate Riemann solvers, parameter vectors, and difference schemes," *J. Comput. Phys.*, Vol. 43, No. 2, 1981, pp. 357–372.

[11]Peraire, J. and Persson, P.-O., "The compact discontinuous Galerkin (CDG) method for elliptic problems," *SIAM J. Sci. Comput.*, Vol. 30, No. 4, 2008, pp. 1806–1824.

[12]Bonet, J. and Wood, R. D., *Nonlinear continuum mechanics for finite element analysis*, Cambridge University Press, Cambridge, 1997.

[13]Persson, P.-O. and Peraire, J., "Curved Mesh Generation and Mesh Refinement using Lagrangian Solid Mechanics," *47th AIAA Aerospace Sciences Meeting and Exhibit, Orlando, Florida*, 2009, AIAA-2009-949.