European Workshop on High Order Nonlinear Numerical Methods
for Evolutionary PDEs: Theory and Applications (HONOM 2013)
Bordeaux, France, March 18-20, 2013

# AN ADVANCED IMPLICIT MULTI-STEP METHOD APPLIED TO THE DISCONTINUOUS GALERKIN DISTRETIZED NAVIER-STOKES EQUATIONS

**A. Nigro[1], C. De Bartolo[1], F. Bassi[2] , A. Ghidoni[3]**

[1] University of Calabria-Department of Mechanical Engineering
Ponte P. Bucci cubo 44/C, 87036 - Rende (CS), Italy
anigro@unical.it, c.debartolo@unical.it

[2] University of Bergamo-Department of Industrial Engineering
Viale Marconi 5, 24044 - Dalmine (BG), Italy
francesco.bassi@unibg.it

[3] University of Brescia-Department of Mechanical and Industrial Engineering
Via Branze 38, 25123 - Brescia (BS), Italy
antonio.ghidoni@unibs.it

**Keywords**: High-order, unsteady problems, Discontinuous Galerkin finite element method, Two Implicit Advanced Step-point.

## ABSTRACT

In recent years the increasing attention to high-order spatial discretization schemes put forward the development of high-order temporal methods to perform very accurate simulations of unsteady flows. The aim of this paper is to investigate and evaluate multi-step methods that are an enhanced evolution of the more common Backward Differentiation Formulae (BDF), collectively named in [1] Implicit Advanced Step-point (IAS) methods. Among them, the Two Implicit Advanced Step-point (TIAS) method has the best stability properties being A-stable with order up to 6. The general TIAS algorithm involves four stages: the first three are predictor stages that use a standard $k$-step BDF scheme, the last one is a corrector stage that uses an advanced implicit $k$-step formula of order $k + 1$. The TIAS method was presented in [1–3], where the focus was to describe the theoretical development and the mathematical framework of this new technology without any numerical application. In this work we present numerical results obtained using this advanced multi-step method applied to a high-order Discontinuous Galerkin (DG) discretization of the Navier-Stokes equations [4].

The performance of the DG-TIAS scheme has been evaluated by means of two test cases: an inviscid isentropic convecting vortex and a laminar vortex shedding behind a circular cylinder. The first test case, for which an exact solution is available, aims at assessing the accuracy and the order of convergence of the scheme by performing a temporal refinement study. The second test case aims at evaluating the potential of this approach for more realistic problems. To clearly illustrate the advantages of the high-order time discretization, the results of the sixth-order accurate TIAS scheme (TIAS6) will be

compared with the results obtained with the standard second-order accurate BDF scheme (BDF2) using the same fourth-order accurate spatial discretization (P3 elements). Fig. 1 shows the numerical pressure profiles for the inviscid test case after one period $T$ of vortex revolution using the P3-BDF2 and P3-TIAS6 schemes. Fig. 2 shows the snapshots of the velocity magnitude field for the unsteady vortex shedding behind a circular cylinder computed with the P3-BDF2 (top row) and the P3-TIAS6 (bottom row) schemes for a fixed time step $\Delta t \simeq T/10$, where $T$ is the vortex shedding period.

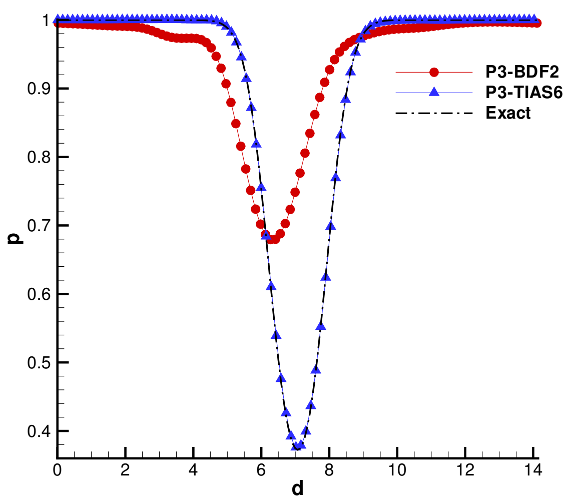


Figure 1: Comparison of the computed pressure profiles with respect to the exact solution along the diagonal $d$ of the grid for $\Delta t = T/40$.

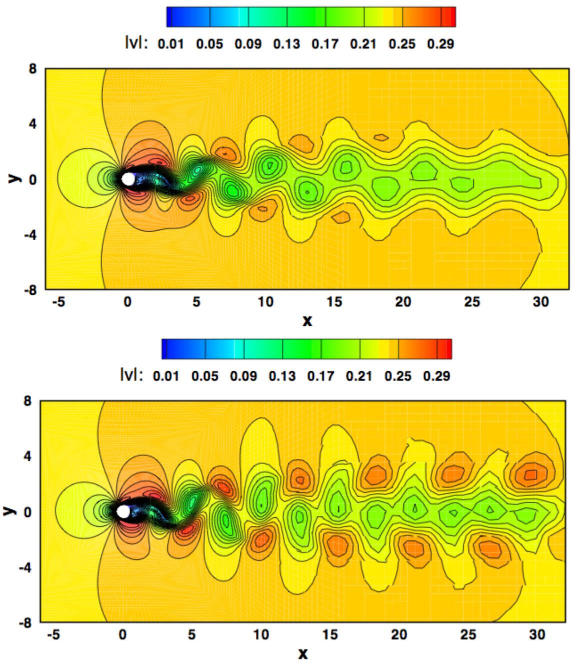


Figure 2: Snapshots of the velocity magnitude field for $\Delta t \simeq T/10$. Top row: P3-BDF2, bottom row: P3-TIAS6.

# References


[1] G. Psihoyios: A general formula for the stability functions of a group of Implicit Advanced Step-point (IAS) methods. *Mathematical and Computer Modelling*, (Elsevier, ISSN 08957177), vol. 46 (no.1-2), pp. 214-224, 2007.

[2] G. Psihoyios: Advanced Step-point Methods for the Solution of Initial Value Problems, *PhD Thesis*, University of London - Imperial College of Science and Technology, 29/12/1995.

[3] G.Y. Psihoyios and J.R. Cash: A stability result for general linear methods with characteristic function having real poles only. *BIT Numerical Mathematics*, (Springer, ISSN 00063835), vol. 38 (no. 3), pp. 612-617, 1998.

[4] F. Bassi and S. Rebay: GMRES discontinuous Galerkin solution of the compressible Navier-Stokes equations. *Lecture Notes in Computational Science and Engineering*, vol. 11, pp. 197-208, Springer-Verlag, Berlin, 2000.