Highly accurate surface and volume integration for level set methods using a moment-fitting approach

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Abstract

We present methods for the numerical integration of functions over domains that are defined by the zero iso-contour of a level set function. Such integrals appear in many contexts, but have become a major area of interest due to the recent rise of sharp-interface methods that are trying to resolve local effects with sub-cell accuracy. Examples include include the eXtended Finite Element Method (XFEM), the Finite Cell Method and the Discontinuous Galerkin Method. In a nutshell, such methods share the property that part of the burden of discretization is shifted to the numerical integration of generic functions over complicated sub-domains for which conventional quadrature are not available. The performance of these methods is thus directly linked to the affordable integration accuracy.

A number of measures to cope with this issue have been proposed in literature. Typically, they are either based on a regularization of discontinuous/singular integrands (e.g., [2]) or an adaptive subdivision of cells intersected by the interface (e.g., [3, 5]). Both approaches reach second-order convergence rates at best, which, especially in case of higher order methods, is often insufficient.

Recently, a method based on the optimization of the non-linear *moment-fitting* equations

$$\begin{pmatrix} \int f_1(\mathbf{x})\, d\mathbf{x} \\ \vdots \\ \int f_M(\mathbf{x})\, d\mathbf{x} \end{pmatrix} = \begin{pmatrix} f_1(\mathbf{x}_1) & \dots & f_1(\mathbf{x}_N) \\ \vdots & \ddots & \vdots \\ f_M(\mathbf{x}_1) & \dots & f_M(\mathbf{x}_N) \end{pmatrix} \begin{pmatrix} w_1 \\ \vdots \\ w_N \end{pmatrix}$$

with given functions $f_1, \dots, f_M$, quadrature nodes $\mathbf{x}_1, \dots, \mathbf{x}_N$ and quadrature weights $w_1, \dots, w_N$ has been developed [1]. If the left-hand side can be obtained, it can be used for the pre-calculation of extremely efficient quadrature rules on very general domains. In [6], the authors restrict themselves to piecewise linear interfaces, simplify the left-hand side and show how the non-linear optimization process can be reduced to a linear one in this context. The proposed modifications render the approach practicable in a broader range of applications comprising moving level sets. Still, the restriction to piecewise linear interface approximations makes a refinement near curved boundaries inevitable and, if high accuracy is required, the process becomes inefficient.

The present work aims at removing the mentioned drawbacks without necessitating a complex, non-general higher order interface reconstruction. On the contrary, the presented methods allow for a realization of the same error levels under very general conditions [4]. In addition, it is easy to implement, reaches extremely high convergence rates and is completely independent of the underlying grid type (see Figure 1).

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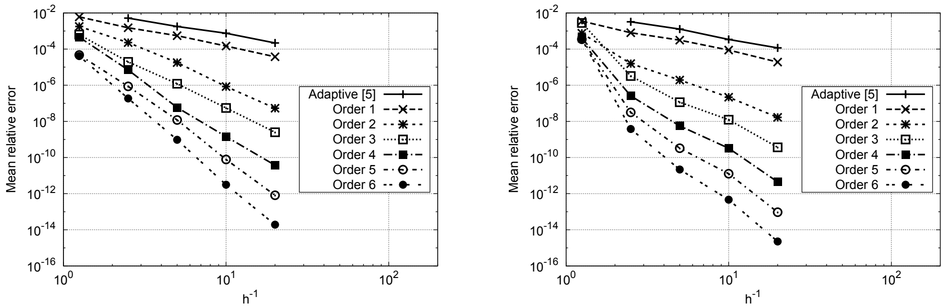

(a) Structured rectangular grid (b) Unstructured triangular grid

Figure 1: Convergence study for the calculation of the arc-length of an ellipse

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