Abstract for HONOM 2013

Analysis of discrete shock profiles of high-order compact schemes

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Shock capturing is a key point in the application of high-order numerical schemes to compressible flows. In this respect, Residual-Based Compact schemes (RBC) seem to be good candidates as they can reasonably capture shock waves without requiring special treatment such as artificial viscosity, numerical filters or limiters, see *e.g.* [1-5]. The characteristic feature of RBC schemes is to be expressed only in terms of approximation of the exact residual, *i.e.* the sum of all the terms in the governing equations. They are written as a centered consistent part, plus a dissipation term constructed from first derivatives of the residual. Their spatial accuracy is of order q, a odd number so that dissipative errors dominate dispersive ones. Scheme compactness allows to reach order q=3 on a 3-point stencil (in each space-direction) and order q= 5 or 7 on a 5-point stencil.

The present study aims to better understand the shock capturing capabilities of high-order schemes through an analysis of the discrete shock profiles of RBC schemes. Several important mathematical works have been devoted to the study of existence and stability of shock profiles for low-order schemes, see *e.g.* [6-10]. Surprisingly, we have found that the discrete profiles of the RBC schemes of order q=3, 5 and 7 can be analytically determined for steady shocks, without linearizing. This is unexpected since an exact discrete shock solution usually cannot be obtained, even for the simplest methods such as the Lax-Friedrichs scheme or the Lax-Wendroff scheme.

In particular, the discrete shock solutions of the RBC schemes will be described for the inviscid Bürgers equation taking into account the boundary conditions, the treatment of the first interior point on both sides for 5-point schemes and the matching of the supersonic and subsonic zones. From these solutions, we will discuss the oscillatory nature of the shock profiles depending on the order of the scheme and on the location of the shock in the mesh. The theoretical results will be confirmed by numerical experiments.

Another aspect of the work will concern the so-called modified or equivalent equation. For RBC shock profiles, it is possible to integrate this high-order differential equation and make comparisons with the exact solution of the discrete scheme. This can provide valuable information on the validity of the modified equation approach for shock problems solved with high-order schemes. This may be important for future work because the modified equation is generally simpler to study than the discrete scheme.

References:

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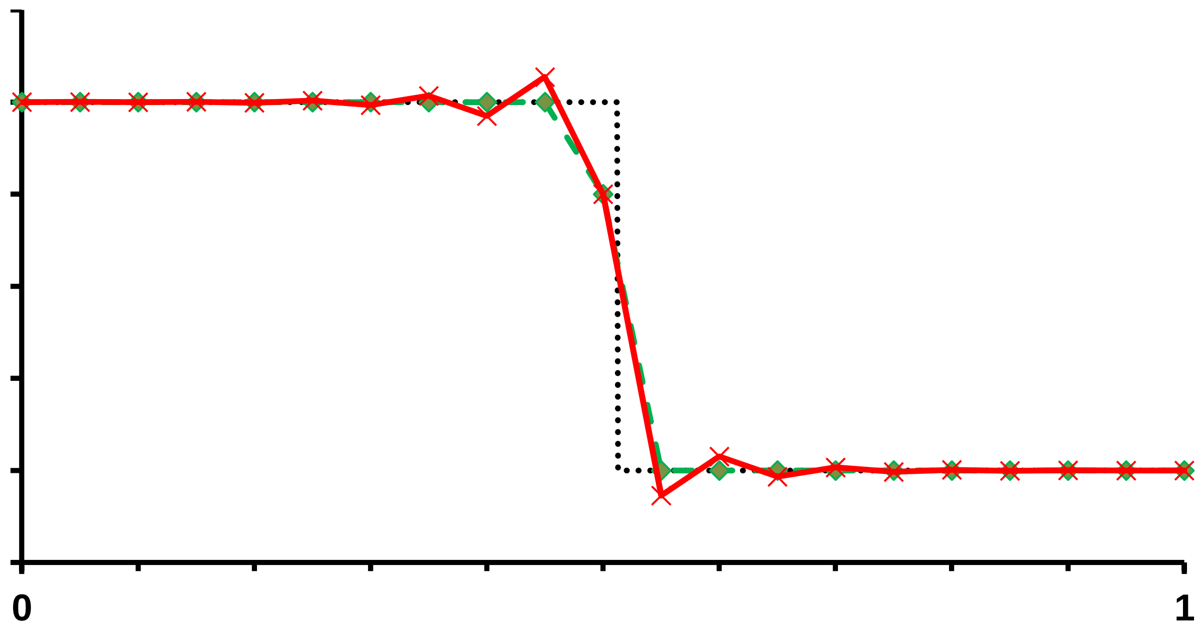
Example of result:



Solution of the exact problem (dotted line) and exact solution of RBC3 (dashed line) and RBC7 (solid line). Exact shock is located at x = 0.5 + δx/4