A high order realizability preserving Discontinuous Galerkin method for Eulerian simulation of spray transport on unstructured meshes

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Abstract

The present contribution focuses on the transport of a cloud of discrete dense inclusions, liquid or solid, within a continuous fluid phase. Among the large range of applications, we particularly focus here on the domain of multiphase combustion. In this context, the particle repartition is represented by a Number Density Function (NDF) whose variations are governed by an evolutionary PDE, the Williams-Boltzmann equation [6], in the so-called *phase space*. The phase variables are: the space-time position, the droplet size, the velocity and the temperature. Because the phase space is high-dimensional, the direct resolution of the Williams-Boltzmann equations is hardly considered, and the used Eulerian methods rather solve for a finite set of moments, which are integrated quantities over the phase space. Following the modeling hypothesis, the evolution of the moments is ruled by a system of conservation law which can be either hyperbolic or weakly hyperbolic, in the sense that the Jacobian matrix eigenvalues are all real and equal and the system is no more diagonalizable. In this context, it is known [1] that the solutions of such a system can become non-measurable in finite time, as for the δ-shock which is an infinite accretion of sticky particles at a point.

Previous effort [4] has been done to integrate properly the system of moments on structured hexahedral grids with a second order least diffusive kinetic scheme. This scheme provides TVD properties, as well as numerical fluxes exact integration in time. However, realistic applications as combustion chambers having complex geometries, an extension to unstructured meshes has to be studied and this contribution presents a possible and promising approach.

At EM2C, the development of new high order schemes for unstructured grids has become an autonomous axis of research. Our strategy first consists in a review of existing schemes, to get a clear picture of the available solutions. This review is based on the following requirements on the numerical scheme: it must work on unstructured grids at high order and must fulfill the key *realizability condition*. A set of moments $\mathbf{W}$ is called realizable when there exists a NDF whose first moments are $\mathbf{W}$. This defines a realizability space $\mathcal{S}$, in which the numerical states have to stay, and $\mathcal{S}$ is known to be convex [3]. Central schemes and more generally finite element approaches, (*e.g.* Two-Step Taylor-Galerkin, [2]), require a stabilization procedure such as artificial viscosity, whose set up is rather empirical. On the other hand, high order finite volume methods, such as ENO/WENO schemes, provide upwinding properties and can theoretically preserve realizability, but at the price of severe loss of accuracy at extrema and larger computational stencils.

To circumvent these limitations, Discontinuous Galerkin schemes appear to be a promising solution, mainly because of the high modularity of these schemes. Using the ideas of [7], we set an arbitrary high order Discontinuous Galerkin framework which expresses the update of the mean state $\mathbf{W}_i$ of each element as a convex combination of the reconstructed states at the quadrature points. A generic polynomial projection keeps the quadrature states within $\mathcal{S}$ while maintaining the global high order accuracy. $\mathcal{S}$ being convex, the realizability of the mean state $\bar{W}_i$ in each element C_i of the mesh is preserved. The assessment of this method has been the subject of our project at the latest CTRSummer Program, [5]. We have studied one and two dimensional Pressureless Gas Dynamics (PGD) problems, with strong accumulations (δ-shocks) coexisting with vacuum generating nearby regions, see Figure 1. For the sake of completeness, the current study also wishes to compare the efficiency of the above method with other possible discretizations on unstructured grids known in the literature. One can in particular cite the TTCG scheme, currently used

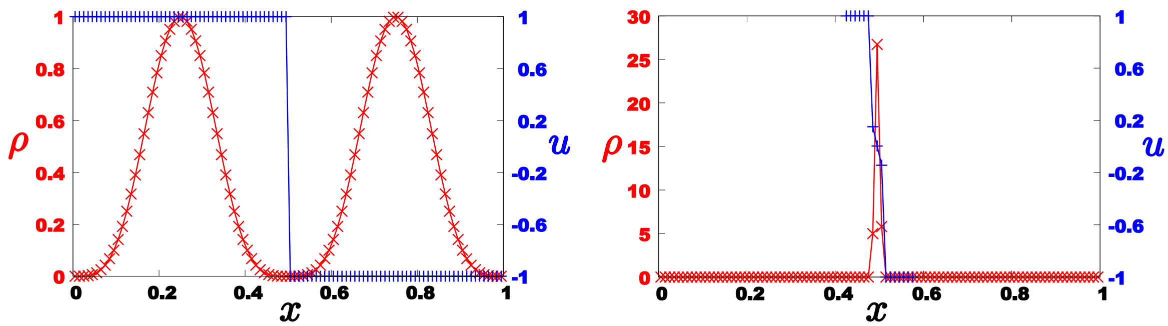

Figure 1: Snapshots of the solution of a PGD problem with C^∞ initial conditions. Left: Initial solution, ρ and $u = \frac{\rho u}{\rho}$. Right: ρ and u at $t = 0.5$, third order DG.

in the semi-industrial code AVBP, and the extension of the finite volume kinetic scheme to higher order reconstruction on triangular grids, [4].

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References

[1] F. Bouchut. On zero pressure gas dynamics. In *Advances in kinetic theory and computing*, volume 22 of *Ser. Adv. Math. Appl. Sci.*, pages 171–190. World Sci. Publ., River Edge, NJ, 1994.

[2] O. Colin and M. Rudgyard. Development of high-order taylor-galerkin schemes for LES. *Journal of Computationnal Physics*, 162(2):338–371, 2000.

[3] H. Dette and W. J. Studden. *The theory of canonical moments with applications in statistics, probability, and analysis*. John Wiley & Sons Inc., New York, 1997.

[4] D. Kah, F. Laurent, M. Massot, and S. Jay. A high order moment method simulating evaporation and advection of a polydisperse spray. *J. Comput. Phys.*, 231(2):394–422, 2012.

[5] A. Larat, M. Massot, and A. Vié. A stable, robust and high order accurate numerical method for eulerian simulation of spray and particle transport on unstructured meshes. In *Proceedings of the summer program 2012, Center for Turbulence Research, Stanford University*, pages 1–10, 2012.

[6] F.A. Williams. Spray combustion and atomization. *Physics of Fluids*, 1:541–545, 1958.

[7] Xiangxiong Zhang, Yinhua Xia, and Chi-Wang Shu. Maximum-principle-satisfying and positivity-preserving high order discontinuous galerkin schemes for conservation laws on triangular meshes. *J. Sci. Comput.*, 50(1):29–62, January 2012.