Runge-Kutta Residual Distribution Schemes

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Residual distribution schemes have been proposed as alternatives to the more commonly-used finite volume and finite element approaches to approximating conservation laws and purport to provide a more natural framework within which to incorporate genuinely multidimensional flow physics. They are capable of providing simulations which are both free of unphysical oscillations and second order accurate, even in the presence of smooth turning points in the solution, for both steady state and time-dependent problems.

As the steady-state residual distribution framework matured, the focus of development shifted towards time-dependent problems, and schemes with higher than second order accuracy. This additional spatial accuracy has been obtained by evaluating the residual within each mesh element from a local higher order interpolant of the dependent variables and distributing it in a linearity preserving manner [2]. The extra information required to construct this interpolant on a mesh element may be provided by reconstructing derivatives at its vertices from the local solution values [4], or explicitly assigning additional discrete solution values to the element, either via uniform subdivision of the mesh [3] or the extension of the stencil beyond the element [6]. Higher order accuracy in time can be achieved using either multistep [1, 4] or multistage [6] approximations of the time derivative.

This paper is concerned with the development of multistage residual distribution schemes, and particularly the possibility of obtaining algorithms which combine positivity (freedom from unphysical oscillations) with higher than second order accurate in both space and time. A framework for Runge-Kutta residual distribution will be presented and the inversion of the mass matrix relating to the integral of the time derivative will be discussed. An approach will be described which combines a "limited" quadratic interpolant, which has already been demonstrated to give positivity and higher than second order accuracy in the steady state case [5], with a technique for manipulating the mass matrix to ensure that its inversion does not cause loss of positivity in the time-dependent case. Preliminary results will be shown for scalar, multidimensional, conservation laws on unstructured meshes.

References

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