On the Accuracy of Boundary Layer Grids for Discontinuous Galerkin based Discretizations

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Abstract

When discretizing boundary layers, it is common to use grid stretching to increase the resolution close to the wall. Using the discontinuous Galerkin (DG) approach for the spatial discretization, e.g. [2, 4, 3, 5], there are typically two steps involved in defining the approximation space: the first step is to decompose the domain into suitable sub-domains (grid cells or elements); the second step is to choose the degree N of the local polynomial ansatz for each sub-domain and the type of polynomial basis function (e.g. nodal or modal). Compared to most other methods such as Finite Difference (FD) and Finite Volume (FV) schemes, the grids of high order DG discretizations are coarser due to the polynomial induced sub-cell resolution. Figure 1 shows a typical boundary layer mesh for a FD/FV based discretization and a comparable high order DG mesh, which is coarser by a factor of four.

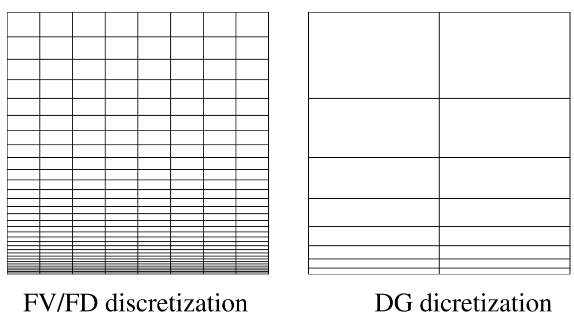

Figure 1: Typical illustrative boundary layer meshes for FV/FD and DG.

It is evident from this plot, that the stretching of the coarse DG mesh is weaker, leading to a lower spatial resolution near the wall. Typically, the internal mapping of the grid cell is linear. In this case, a higher spatial wall resolution can only be achieved by a high order polynomial.

We propose that the definition of the DG approximation space involves three building blocks instead of two: the physical mesh, the local polynomial ansatz *and* the local grid cell mapping. We will show that when using boundary layer meshes, the accuracy can be improved by including a suitable local grid cell mapping without actually changing the physical dimensions of the grid cell. The approach can be seen as a sort of r-adaptivity inside the grid cells. The positive effect on the accuracy was already shown for single domain spectral methods [1, 7], where r-adaptivity is the only way to better resolve local features of the solution, when keeping the number of solution points constant. The accuracy study is based on the stationary solution of a model boundary layer problem, the singular perturbation problem (SPP) [6, 7].

References

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