Finite volume formulation of a third-order residual-based compact scheme for unsteady flow computations

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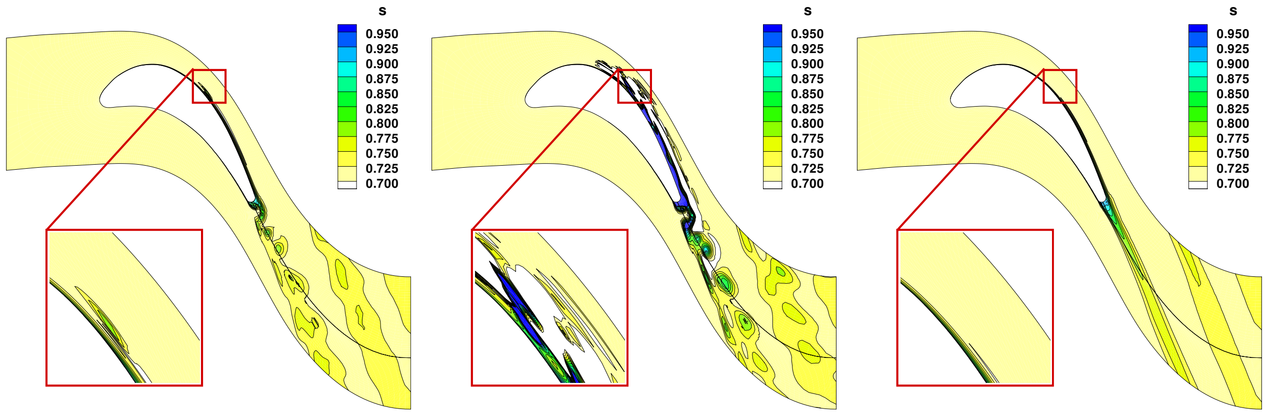
Abstract: The paper illustrates design principles and a comprehensive study of a Cauchy-stable Finite Volume (FV) Residual-Based scheme of third-order accuracy for unsteady compressible flows on structured grids. The scheme uses weighted discretization coefficients to preserve high-accuracy on deformed grids.

Keywords: Residual based scheme, High-order, Unsteady, Curvilinear mesh

Classical methods for calculating compressible flows on a structured mesh rely on a directional approach in which space derivatives are approximated independently direction by direction. In the present work, we study compact approximations that provide a high accuracy not for each space derivatives treated apart but for the complete residual r, *i.e.* the sum of all the terms in the governing equations. For unsteady problems, r also includes the time derivative. Schemes of this type are said to be Residual-Based Compact (RBC) and have been developed in the last ten years ([1, 2, 3] for instance) with application to realistic flow configurations in aerodynamics and aeroacoustics.
The finite-volume (FV) method is widely used in computational fluid dynamics for its ability to handle complex geometries while ensuring conservation. It consists in discretizing a system of conservation laws from its integral form over a mesh composed of small volumes or cells. Point-wise cell-centered FV RBC schemes for steady flows have been developed in the case of both structured [5] and unstructured meshes [4]. For steady problems, the residual does not contain time derivatives, which simplifies accuracy and stability analysis. For unsteady problems, this kind of analysis is no longer sufficient, and straightforward extensions of the steady schemes to unsteady flows may lead to numerical instabilities for some flow conditions [3]. In this work we design a dissipative FV RBC scheme for unsteady compressible flows. We call this scheme RBCi where the i stands for *irregular* since it is designed for irregular meshes. Since we are essentially interested in spatial accuracy properties, the study restrict our discussion to semi-discrete schemes. Considering a structured mesh composed of hexaedral cells $\Omega_{j,k,l}$which boundary is $\partial\Omega_{j,k,l}$, we define a FV residual $R_{j,k,l}$ as:

$$R_{j,k,l} := \frac{d}{dt}\int_{\Omega_{j,k,l}} w \; d\Omega + \sum_{\Gamma\in\partial\Omega_{j,k,l}} \int_{\Gamma} \boldsymbol{\phi}.\boldsymbol{n} \; d\Gamma \qquad (1)$$

where t is the time, w is the vector of conservative variables, $\boldsymbol{\phi}$ is the physical flux density $\boldsymbol{\phi} = [f, g, e]$ and $\boldsymbol{n}$ is the unit outward normal. The RBCi formulation is developed, similarly to finite difference RBC schemes [3], in terms of a main residual, used to construct the consistent part of the scheme, and mid-point residuals, used to define a residual-based dissipation. These residuals are approximated in the FV framework using weighted discretization operators, such that RBCi is third-order on smoothly distorted meshes and second-order accurate on generic structured meshes. The dissipation operator of RBCi is shown to be dissipative for any flow condition, on Cartesian meshes, leading to a Cauchy-stable discretization. A study of the numerical spectra associated with the RBCi spatial discretizations is carried out and strengthens the previous results.



(a) RBCi. (b) RBC3. (c) RBC2.

Figure 1: Snapshot of the unsteady entropy field.

The resulting scheme is implemented within the industrial code elsA developed by the Numerical Simulation and Aeroacoustics Department (DSNA) of ONERA, which allows for solving the Euler or Reynolds-Averaged Navier-Stokes equations in a multi-block FV framework.

The case of the advection of an inviscid vortex confirms the accuracy order of the RBCi and its properties on randomly shaken meshes. Then the scheme is applied to the laminar flow past a cylinder and the VKI LS-59 turbine cascade [6]. Full details of these test cases will be provided at the conference. The latter case has been proven to be an excellent testing bench to study both the resolvability properties and robusteness of numerical schemes since it is a high-loaded rotor blade with a thick, rounded trailing edge originally designed for near-sonic exit flow conditions. Morover, the case has been computed on a 384x32 mesh with a conformal join which results in a highly distorted mesh given the geometry. Preliminary results for this problem are shown in Fig. 1. RBCi captures vortex shedding, unlike the second-order RBC scheme which predicts a steady wake. Moreover, thanks to the weighted discretization operators, RBCi provides an accurate and non oscillatory solution, while a direct extension of the FD RBC scheme to curvilinear meshes (RBC3 scheme) generates unphysical spurious oscillations in the region where the computational mesh is highly distorted (*cf*. Fig. 1).

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