Implicit High Order Residual Distribution Scheme for Three Dimensional Compressible Turbulent Flows

R. Abgrall[†], D. De Santis[†], M. Ricchiuto[†]

[†]Team BACCHUS, INRIA Bordeaux Sud Ouest and Institut de Mathématiques
200 avenue de la Vieille Tour, 33405 Talence Cedex, France

[†] Address
{remi.abgrall, dante.de_santis, mario.ricchiuto}@inria.fr

ABSTRACT

Residual Distribution schemes represent an interesting alternative to Discontinuous Galerkin (DG) schemes [1]. While computationally compact and probably more flexible, DG schemes suffer from the serious drawback of a very fast growth of the number of degrees of freedom with the cell polynomial degree. In RD schemes the formulation remains local, as in DG, but the number of degrees of freedom grows less quickly. The price to pay is to impose a continuous approximation of the solution, even though some papers report their extension to discontinuous approximation. [2, 3]

The RD discretization of the diffusive terms involves the reconstruction of the gradient of the solution in order to approximate continuously the viscous fluxes along the faces of the elements. It has recently showed that, in order to preserve the overall accuracy of the numerical scheme, the gradients of the solution must be computed with the same order of accuracy used for the solution approximation. This point is particularly important when advection and diffusion have the same order of magnitude.

The proposed scheme for the discretization of the Navier-Stokes equations remains compact and preserves the formal accuracy of method on all the range of the Reynolds number. The scheme is very flexible being able to use hybrid unstructured grids in two or three dimensions, with higher order (> 2) discretization. Furthermore curvilinear elements are employed in order to reduced the error due to surface discretization.

Turbulent phenomena are taking into account by the means of RANS modeling, in particular the Spalart-Allmaras turbulence model is used in a fully coupled approach with the RANS equations. Modifications of the Spalart-Allmaras equation are introduced respect to its original formulation in order to make the RANS systems of equation more robust.

A particular emphasis is also put on the implicit formulation of the numerical scheme, in particular the main goal is the construction of a robust and effective implicit method which is able to reach the steady state solution, up to machine zero approximation, in an efficient way. The use of the matrix-free [6, 4] approach and the more recent non-linear LU-SGS [5] method are investigated.

The effectiveness of the proposed approach is demonstrated in the computation of turbulent test cases. Two and three dimensional complex configurations are considering and the efficiency of the higher order discretization is compared against that of second order methods.

References

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