

The Multi-dimensional Optimal Order Detection (MOOD) method

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First introduced in [1,2], the Multi-dimensional Optimal Order Detection (MOOD) method proposes a new strategy to provide up to sixth-order approximations to hyperbolic scalar or vectorial problems for two- and three-dimensional geometry with unstructured meshes. The issue addressed in [3] was to extend the MOOD method to three-dimensional geometries with general polyhedral unstructured meshes for the scalar advection equation and the hydrodynamics Euler system.

The method casts in the generic framework of the finite volume method but fundamentally differs from the traditional techniques by the specific detection-limitation procedure implemented by the authors. Indeed, classical high-order polynomial reconstruction-based schemes such as MUSCL or ENO/WENO methods rely on an *a priori* limiting procedure to achieve some stability properties. For instance, in MUSCL-like methods unlimited slopes are reduced through the use of limiters to respect some Discrete Maximum Principle or Total Variation Diminishing properties. In the same way, ENO/WENO-like methods employ an essentially non-oscillatory polynomial which provides an accurate solution while preventing undesirable oscillations from appearing.

We state that such limitation strategies are *a priori* in the sense that only the data at time t^n are used to first perform the limitation procedure and then compute an approximation at time t^{n+1} . Generally, the “worst case scenario” (speculative approach) has to be considered as plausible and, consequently a “precautionary principle” is applied. It results that most of the time the limitation mechanism unnecessarily operates and may reduce the scheme accuracy due to restrictive assessments.

The MOOD principle lies in a different approach (objective approach) since we first compute a candidate solution for time t^{n+1} and use this *a posteriori* information to check if the proposed approximation is valid. Roughly speaking, we compute a candidate solution without any limitation using local polynomial reconstructions to provide accurate approximation of the flux (the degree is set to a prescribed maximal value). We then detect if this solution locally fails to fulfill some stability criteria (detection of problematic cells) and further decrement polynomial degree only on problematic regions (limitation step) before recomputing a new candidate solution. An iterative procedure (the MOOD algorithm) is carried out by successively decrementing the degree to provide the *optimal* local polynomial reconstruction for each cell to satisfy the given stability criteria. At the end of the MOOD algorithm, the candidate solution is eligible and turns out to be the approximation at time t^{n+1} . The *a posteriori* strategy brings new benefits. We dramatically reduce the number of polynomial reconstructions : our technique only requires one polynomial function for each cell. Most of the time, the polynomial with maximal degree is employed since the limitation mechanism is only activated for problematic cells (objective approach). From a physical point of view, the positivity preserving property (for the Euler equations as instance) is simply guaranteed by the *a posteriori* strategy applying a simple detection procedure which checks the physical admissibility of the solution.

Our recent investigations deal with the implicit version of a MOOD scheme and the application of these concepts to solve steady-state solution.

After presenting the basics of MOOD scheme on advection equation and Euler system, we will focus on the specific modification of MOOD to solve steady-state solutions via an explicit and implicit

version of the scheme. Both approaches require some modifications of the MOOD concepts. Next these schemes are implemented and tested. We will present the results of these MOOD schemes on a test suite to assess the validity of such an approach.

Bibliography

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