Numerical simulation of powder-snow avalanches by an hybrid finite volume/finite element method

Caterina CALGARO * Emmanuel CREUSÉ * Thierry GOUDON †

HONOM 2013
Bordeaux, March 18-22, 2013

In this work, we are interested in the simulation of powder-snow avalanches, by the numerical approximation of a particular so-called Kazhikov-Smagulov model given in its non-dimensional form by :

$$\left\{\begin{array}{rcl} \partial_t\,\rho + \nabla_x\cdot(\rho v) &=& \dfrac{1}{\mathrm{Re}\,\mathrm{Sc}}\Delta_x\,\rho, \\ \rho\big(\partial_t\,v + (v\cdot\nabla_x)v\big) + \nabla_x p &=& \dfrac{1}{\mathrm{Fr}^2}\rho g + \dfrac{1}{\mathrm{Re}}\mathrm{Div}_x(\mu(\rho)\mathbb{D}(v)) \\ && +\dfrac{1}{\mathrm{Re}\,\mathrm{Sc}}(\nabla_x v - \nabla_x v^T)\nabla_x\rho, \\ \nabla_x\cdot v &=& 0. \end{array}\right.$$

Here, ρ, v and p are respectively the density, the velocity and the pressure of the fluid, and g stands for the unit vector pointing in the direction of the gravity field. The involved physics is embodied into three dimensionless parameters: the Reynolds number Re, the Froude number Fr and the Schmidt number Sc. Considering L, U, ν and κ as a characteristic length, velocity, viscosity and mass diffusivity, they are respectively defined as :

$$\mathrm{Re} = \frac{\mathrm{U\,L}}{\nu}, \qquad \mathrm{Fr} = \frac{\mathrm{U}}{\sqrt{\|g\|\mathrm{L}}}, \qquad \mathrm{Sc} = \frac{\nu}{\kappa}.$$

*Université Lille 1 - Laboratoire Paul Painlevé and E.P.I. SIMPAF - INRIA Lille Nord Europe, Cité scientifique, 59655 Villeneuve d'Ascq Cedex, France. caterina.calgaro@math.univ-lille1.fr, emmanuel.creuse@math.univ-lille1.fr

†E.P.I. COFFEE - INRIA Sophia Antipolis Méditerranée, 2004, route des Lucioles - BP 93, 06902 Sophia Antipolis Cedex & Labo. J. A. Dieudonné, CNRS-Université Nice Sophia Antipolis, UMR 7351, Parc Valrose 06108 Nice cedex 02, France. thierry.goudon@inria.fr

Real avalanches phenomena lead to Reynolds numbers around 10^8 and Froude ones about unity, what constitutes, up to now, an unreachable parameters set both numerically and experimentaly. Nevertheless, simulations as well as laboratory experiences for lower values of Re can be considered. In that case, it can be observed that the larger Re is and the smaller Fr is, the stiffer the problem is.

In this talk, the modeling leading to the above equations will first be recalled, in the context of the modeling of mixture flows. Then, a high order finite volume/finite element scheme will be described, allowing to perform efficient numerical simulations. Finally, several tests corresponding to different characteristic regimes will be presented, commented and compared to the available numerical as well as experimental database.

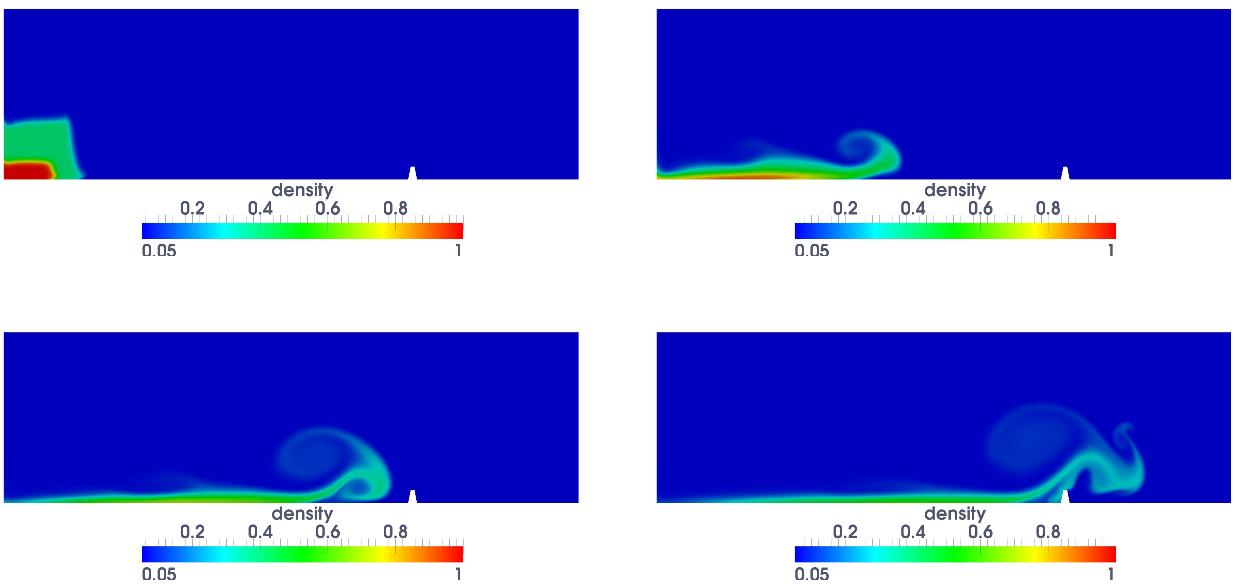

Figure 1: Density at times $t = 1.03, 5.16, 8.26, 10.33$, for Fr = 1.8 and Re = 900, and corresponding to the interaction of an avalanche with an obstacle.

[1] C. Calgaro, E. Creusé, and T. Goudon. An hybrid finite volume-finite element method for variable density incompressible flows. *J. Comput. Phys.*, 227(9):4671–4696, 2008.

[2] C. Calgaro, E. Chane-Kane, E. Creusé, and T. Goudon. L^∞ stability of vertex-based MUSCL finite volume schemes on unstructured grids; simulation of incompressible flows with high density ratios. *J. Comput. Phys.*, 229(17):6027–6046, 2010.