Time–Space Domain Decomposition applied to the Navier–Stokes Equations

Oana Ciobanu, Xavier Juvigny and Juliette Ryan
Corresponding author: oana.ciobanu@onera.fr
ONERA, Chemin de la Hunière et des Joncherettes,
BP 80100, FR-91123, PALAISEAU CEDEX

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In this work, we are interested in applying an implicit time space domain decomposition method to a Navier–Stokes problem in the context of the aeroacoustics which requires high order precision schemes. The purpose of this paper is to analyse and compare the precision of a finite volumes method with that of a DG method on two stiff cases with domain decomposition methods.

Domain decomposition methods split large problems into smaller subproblems that can be treated in parallel. Usually, only space domain decomposition method is used to provide high-performing algorithms in many fields of numerical applications. To achieve full performance of large clusters with up to 100 000 nodes (such as recently the IBM Sequoia, or GPUs) requires another dimension to parallelize. Another essential gain to be obtained from time-space domain decomposition is the ability to apply different time-space discretisation on subdomains thus improving efficiency and convergence of implicit schemes.

The Schwarz Waveform Relaxation (SWR) domain decomposition methods based on [1] and developed in [2] is one method to realize this goal. Initially analyzed for linear problems, it has been extended to nonlinear cases thanks to the Newton scheme. This process can be done in several ways. We choose to use a Newton–Schwarz method as in [3],[4],[5] that consists in applying a global Newton linearization then dividing the linear system in several local overlapping subsystems. Each subsystem is solved in parallel and then the whole solution is rebuilt through proper continuity boundary conditions.

The non linear SWR is applied to the full Navier–Stokes equations with two types of viscous Jacobian, second order litteral approximation as given in [6] and exact jacobian approximated with second order schemes. Space discretisation is achieved with finite volumes on structured grids.

Results are first validated on the viscous motion of a 2D isolated vortex in an uniform free-stream based on H.C. Yee [7].

Finally the case of a 2D low-Reynolds mixing layer where a high precision scheme is required, is studied especially focusing on the acoustic waves emitted by the vortex pairings in a perturbed mixing layer [8]. Thanks to this particularly sensitive case, we are able to compare the precision of our schemes with those of an explicit DG solver developed by L. Halpern, J. Ryan and M. Borrel in [9].

For the Navier–Stokes equations, Space–time Discontinuous Galerkin (DG) discretisation is known to have a good behaviour and provides accuracy and performance. This paper shows that comparable performances may be found with a simple implicit finite volumes method thanks to the introduction of SWR.

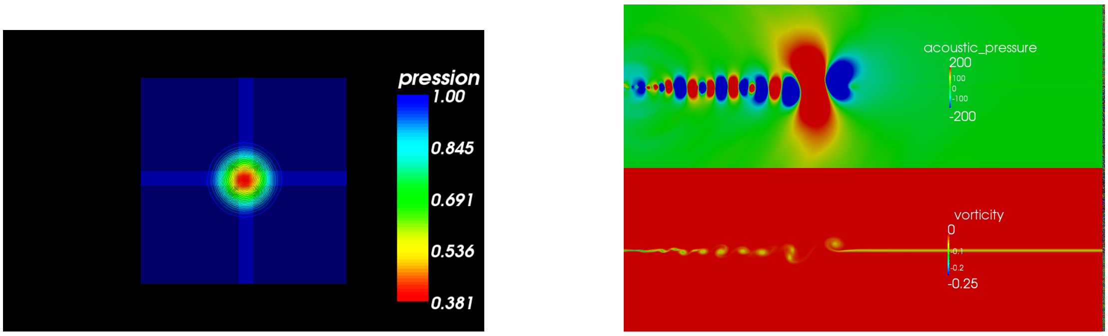

Figure 1: (left) 2D vortex , (right) Mixing layer acoustic pressure field.

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