A priori-based mesh adaptation for third-order accurate Euler simulation

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The proposed communication takes place in a set of works dedicated to mesh adaptation in third order accurate CFD calculations. For the applications we consider, the addition of mesh adaptation is paramount for obtaining a third order numerical convergence with moderate mesh sizes. Further, error models show that anisotropic mesh adaptation is compulsory, see for example [1]. In order to build rather easily anisotropic criteria, we choose an a priori analysis. Modern high order methods applying to hyperbolic equations involve distributive schemes, Taylor-Galerkin schemes and ENO ones. In most cases, the third-order versions are exact if the unknown is a second-degree polynomial. The standpoint we adopted is (1) to express the approximation error in terms of quadratic interpolation error and (2) to express this interpolation error in terms of the metric used for defining the mesh.

The 2D approximation scheme we consider is a Central ENO. as introduced in [2] and [3], with a quadratic reconstruction. It is build on the dual cells of a triangulation. When combined with a Riemann solver at interfaces between cells, this approximation involves a stabilizing term, the dominent part of which is a fourth derivative weighted with a local mesh size at the power three. It is then rather dissipative. Further, with dual cells, the number of Gauss quadrature points between cells is high and multiplies the number of Riemann solvers to compute. In the CENO we use, the Riemann solvers are replaced by arithmetic means and a simplified dissipation with sixth derivatives and fifth-order impact on error is added. This brings a lower cpu cost and a lower dissipation.

Then a simplified *a priori* error analysis is developed. We got inspired by the *a posteriori* analysis by [4]. The state equation is denoted:

$$B(u,v) \quad = \quad F(v) \quad \text{(cont.state eq.)}$$

In a goal-oriented context tending to minimize the error:

$$\varepsilon_h = (u - u_0, g),$$

where u_0 denotes the cell mean of the discrete approximation of u,

$$\begin{array}{rcll} B(v, u_0^*) & = & (g, v) & \text{(discrete adjoint eq.)} \\ B(R_p^0 u_0, v) & = & F(v) & \text{(discrete state eq.)} \end{array} \qquad (1)$$

where R_p^0 denotes the reconstruction operator and u_0^* the adjoint state. Let us introduce the following measure of the approximation error:

$$\tilde{\varepsilon}_h = (R_p^0 \pi_0 u - R_p^0 u_0, g),$$

where π_0 holds for replacing a function by its mean on each cell. We obtain an error model of it expressed in terms of the reconstruction error and the adjoint state u_0^*:

$$(g, R_p^0 \pi_0 u - R_p^0 u_0) \approx B(R_p^0 \pi_0 u - u, u_0^*) \qquad (2)$$

Then we need a representation for the quadratic reconstruction error $R^0_p\pi_0 u - u$ in terms of the metric $\mathcal{M}$ parametrizing our mesh.

$$|R^0_p\pi_0 u - u| \;=\; \left(|\frac{\partial^3 u}{\partial\tau_u^3}|^{\frac{2}{p}}(\delta\tau_u^{\mathcal{M}})^2 + |\frac{\partial^3 u}{\partial n_u^3}|^{\frac{2}{p}}(\delta n_u^{\mathcal{M}})^2\right)^{\frac{p}{2}}$$

where τ_u and n_u are two orthogonal directions, τ_u being the directional where the third order derivative of u is the largest. Then a metric optimization can be performed and produce an adapted mesh.

The above analysis with $p = 2$ has been introduced, together with the novel CENO scheme, in the demonstrator described in [5] which produces adapted meshes for both steady and unsteady flows. Steady flows around airfoils and unsteady acoustics propagations will be the basis of a comparison between second-order based and third-order based mesh adaptation.

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References

[1] A. Loseille, A. Dervieux, P.J. Frey and F. Alauzet, “Achievement of global second-order mesh convergence for discontinuous flows with adapted unstructured meshes”, AIAA paper 2007-4186, Miami, FL, USA, June 2007.

[2] T.J. Barth, “Recent developements of high-order k-exact reconstruction on unstructured meshes”, AIAA Paper 93-0668, 1993.

[3] L. Ivan and C. P. T. Groth, High-Order Central ENO Finite-Volume Scheme with Adaptive Mesh Refinement , AIAA Paper 2007-4323, June 2007.

[4] T.J. Barth and M.G. Larson. A-posteriori error estimation for higher order Godunov finite volume methods on unstructured meshes. In Herbin and Kroner, editors, Finite Volumes for Complex Applications III, pages 4163. Hermes Science Pub., London, 2002.

[5] A. Belme, A. Dervieux and F. Alauzet,“Time Accurate Anisotropic Goal-Oriented Mesh Adaptation for Unsteady Flows”, J. Comp. Phys., 2012, Volume 231, Issue 19, 1 August 2012, Pages 6323-6348