A hybrid finite difference-Weno scheme for Large Eddy Simulation of compressible flows

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This work is aimed towards a better understanding of hydrodynamics instabilities that developp at interfaces separating two gases, like the Richtmyer-Meshkov (RM) instability or the Rayleigh-Taylor (RT) instability. This is of interest in a wide variety of applications, for example in the implosion of inertial confinement fusion targets where RM instabilities occur when shock waves interact with material interfaces. Shock tube experiments enable the realization of RM instabilities. A massively parallel code (the Triclade code) has been developed for several years at the CEA to solve turbulent mixing of perfect or stiffened gas in shock-tube applications. It solves the compressible Navier-Stokes equations for cartesian geometry.

As current computational power does not allow direct numerical simulation of flows under interest, large eddy simulation is a good alternative. Apart from the physical modelling issues associated with LES, a significant challenge is the development of the numerical scheme, in particularly for compressible LES. The main difficulty is that contradictory scheme requirements has to be considered. On the one hand, turbulence is better simulated when the numerical method is not dissipative (any numerical dissipation acting as overwhelming the small scales). Besides, the use of high-order accurate schemes is recommended for LES simulation which are (by construction) under-resolved. On the other hand, the presence of shocks imply the use of a shock-capturing type scheme to stabilize the solution. The numerical dissipation of the scheme should be lower than the subgrid-scale dissipation.

This work investigates a high-order hybrid method for the simulation flows involving discontinuities, turbulence and their interaction. The frame is that of poorly resolved LES simulations.

A necessary and preliminary work is concerned with the problem of instabilities in non-dissipative LES of turbulent flows. Indeed, non-linear numerical instabilities are present in the solution of the Navier-Stokes equations even in the absence of shock waves. These instabilities are potentially more important in LES simulations where the dissipative scales are not resolved. Many authors charge these instabilities to aliasing errors due to the presence of the non-linear convective terms in the Navier-Stokes equations [3]. To deal with the stability problem, one approach consists in rewritting the convective terms in a Skew-symmetric form [5]. It results in reduced ampliture of the aliasing errors relative to the divergence and advective forms. Another view point consists in noting that the Skew-symmetric form of convective operators allows the construction of schemes that locally conserve kinetic energy, which acts as a stabilization procedure.

We consider finite-difference schemes of any order of accuracy (up to 10). By discretizing the divergence form of convective operator, the scheme is locally mass, momentum and total energy conservative (which is necessary to compute

weak solutions and insure convergence). By requiring that the discretized Skew-symmetric form is equivalent to a discrete divergence form, we get the discrete local conservation of kinetic energy. We consider the Skew-symmetric form proposed in [6].
Numerical stability tests will be proposed to illustrate the effets of the Skew-symmetric form on the numerical stability.
The Weno procedure is considered to treat shock. Different strategies are investigated, like for example the introduction of an additional stencil (in comparison with the pionnering work of Shu and Balsara [1]) to revert to the central finite-difference scheme in smooth regions [7].
At the end, the hybrid procedure strongly relies on the definition of a shock sensor. The drawback of most existing methods is that they are problem-dependent. We get satisfactory results with the sensor [2] initially developed for Discontinuous Galerkin methods.
Numerical properties of the method (typically accuracy and stability properties) will be illustrates on different tests, among others the simulation of decaying isotropic turbulence with eddy shocklets [4] and the simulatioon of a RM instability.

References

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