# High Order Arbitrary-Lagrangian-Eulerian One-Step WENO Finite Volume Schemes on Unstructured Triangular Meshes

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**Abstract.** In this paper we present a new class of high order accurate Arbitrary-Eulerian-Lagrangian (ALE) one-step WENO finite volume schemes for solving nonlinear hyperbolic systems of conservation laws on moving two dimensional unstructured triangular meshes. A WENO reconstruction algorithm is used to achieve high order accuracy in space and a high order one-step time discretization is achieved by using the local space-time Galerkin predictor. For that purpose, a new element-local weak formulation of the governing PDE is adopted on moving space-time elements. The space-time basis and test functions are obtained considering Lagrange interpolation polynomials passing through a predefined set of nodes. Moreover, a polynomial mapping defined by the same local space-time basis functions as the weak solution of the PDE is used to map the moving physical space-time element onto a space-time reference element. To maintain algorithmic simplicity, the final ALE one-step finite volume scheme uses moving triangular meshes with *straight* edges. This is possible in the ALE framework, which allows a local mesh velocity that is different from the local fluid velocity.

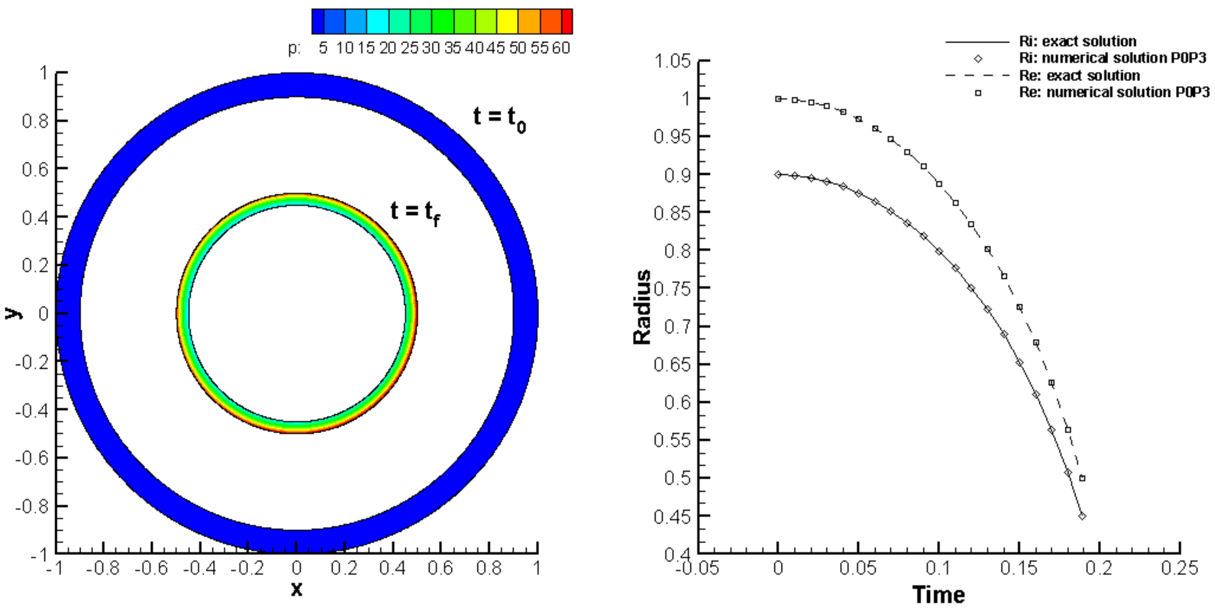


**Figure 1: Kidder problem. Left: Numerical solution at *t*=0.4 and *t*=$t_f$. Right: Evolution of the internal and external radius of the shell and comparison between analytical and numerical solution.**

We present numerical convergence rates for the schemes presented in this paper up to sixth order of accuracy in space and time and show some classical numerical test problems for the two-dimensional Euler equations of compressible gas dynamics, namely the two-dimensional explosion problem, the Kidder problem (Figure 1) and the Saltzmann problem. Finally the proposed numerical scheme has been extended to the *non-conservative* case where the non-conservative products are treated by a path-conservative approach that defines the jump terms on the element boundaries. We apply this new better than second order accurate non-conservative scheme to the full seven--equation Baer--Nunziato model of compressible multi-phase flows in two space dimensions. The use of a Lagrangian approach allows an excellent resolution of the solid contact and the resolution of jumps in the volume fraction (Figure 2).

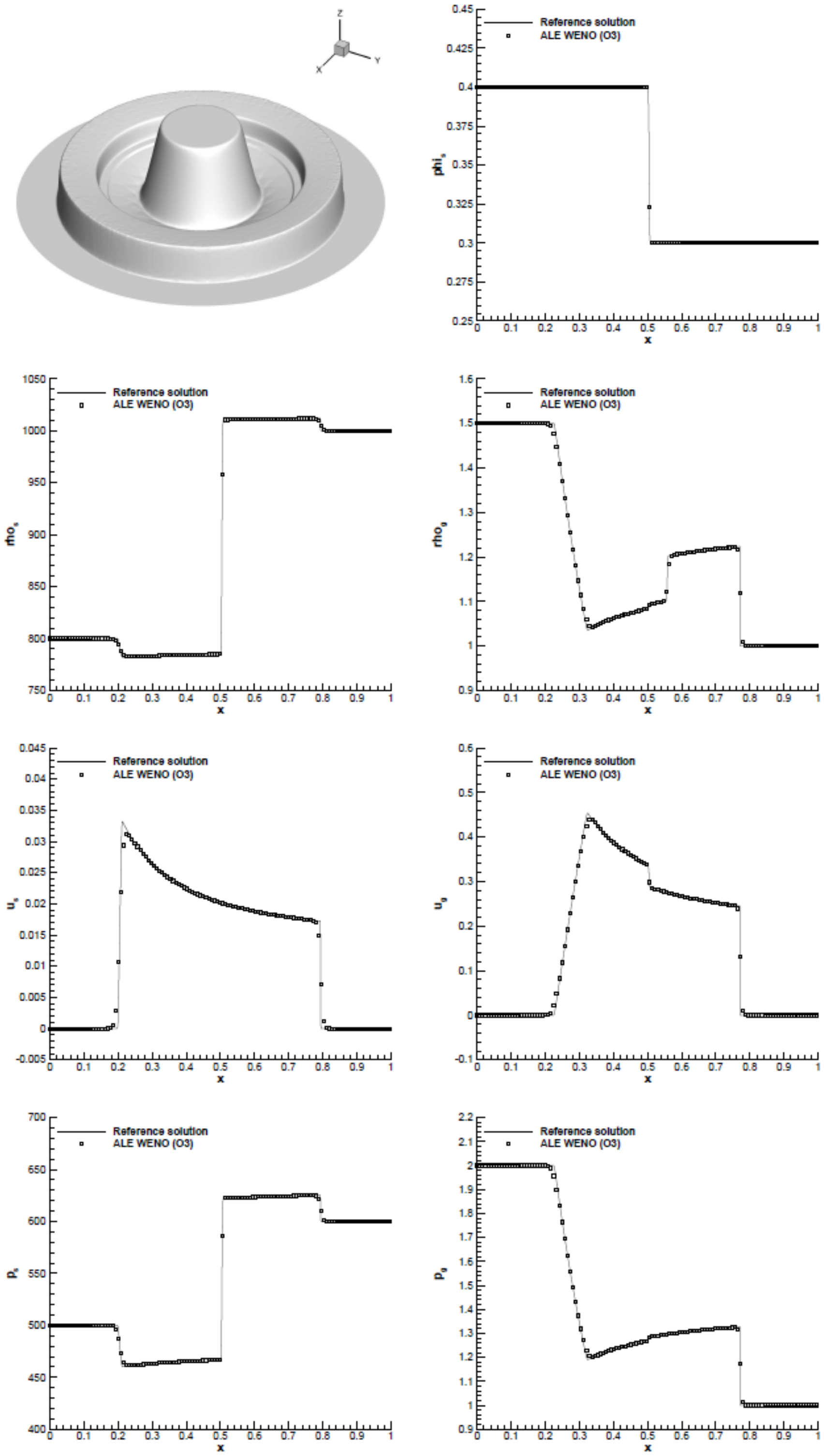


**Figure 2: Results obtained for the first cylindrical explosion problem at $t=0.15$ and comparison with the reference solution.**