A high-order Discontinuous Galerkin discretization for solving two-dimensional Lagrangian hydrodynamics

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The intent of the present work was the development of a high-order discontinuous Galerkin scheme for solving the gas dynamics equations written under total Lagrangian form on two-dimensional un- structured grids. To achieve this goal, a progressive approach has been used to study the inherent numerical difficulties step by step. Thus, discontinuous Galerkin schemes up to the third order of accuracy have firstly been implemented for the one-dimensional and two-dimensional scalar conser- vation laws on unstructured grids. The main feature of the presented DG scheme lies on the use of a polynomial Taylor basis. This particular choice allows in the two-dimensional case to take into general unstructured grids account in a unified framework. In this frame, a vertex-based hierarchical limitation which preserves smooth extrema has been implemented. A generic form of numerical fluxes ensuring the global stability of our semi-discrete discretization in the L2 norm has also been designed. Then, this DG discretization has been applied to the one-dimensional system of conservation laws such as the acoustic system, the shallow-water one and the gas dynamics equations system written in the Lagrangian form. Noticing that the application of the limiting procedure, developed for scalar equations, to the physical variables leads to spurious oscillations, we have described a limiting pro- cedure based on the characteristic variables. In the case of the one-dimensional gas dynamics case, numerical fluxes have been designed so that our semi-discrete DG scheme satisfies a global entropy inequality. Finally, we have applied all the knowledge gathered to the case of the two-dimensional gas dynamics equation written under total Lagrangian form. In this framework, the computational grid is fixed, however one has to follow the time evolution of the Jacobian matrix associated to the Lagrange-Euler flow map, namely the gradient deformation tensor. In the present work, the flow map is discretized by means of continuous mapping, using a finite element basis. This provides an approximation of the deformation gradient tensor which satisfies the important Piola identity. The

discretization of the physical conservation laws for specific volume, momentum and total energy re- lies on a discontinuous Galerkin method. The scheme is built to satisfying exactly the Geometric Conservation Law (GCL). In the case of the third-order scheme, the velocity field being quadratic we allow the geometry to curve. To do so, a Bezier representation is employed to define the mesh edges. We illustrate the robustness and the accuracy of the implemented schemes using several relevant test cases and performing rate convergences analysis.